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An improved lumped capacitance method for dielectric measurement

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An improved lumped capacitance method for dielectric measurement in the broad microwave frequency range is proposed, with which many disadvantages occurring in the previous lumped capacitance method can be avoided. It is an easier and more convenient method for the dielectric measurement of ferroelectrics. Experiments have confirmed the feasibility of the method in the low microwave frequency range.

1. Introduction

Measurement of the complex relative dielectric constant, ε_r, in the microwave frequency range is one of the most important tasks in the investigation of the electrical properties of dielectric materials. However, it is not always easy to complete the measurement accurately. For materials such as ferroelectrics with large anisotropic dielectric constants and very low dielectric loss, there still exist many problems with the measuring methods.

The lumped capacitance method, which employs a coaxial line terminated by a sample-filled capacitor as apparatus, is the most suitable method used for dielectric measurement of ferroelectrics [1]. However, existing treatments involved in the method have some limitations which can lead to inconvenience and sometimes even impossibility in the measurement. These limitations are discussed here and an improved method to overcome the disadvantages is proposed.

2. General review

The lumped capacitance method was first used by Hippel [2], and for several decades many researchers have used it to measure various materials [1, 3–7].

Figs 1a and b show a geometrical sketch of the sample holder and its equivalent circuit for the method, respectively. In these figures, 2a and 2b are the diameters of inner and outer conductors. According to Hippel [2], the relationship between the termination impedance, Z_t (at the plane A–B), the capacitance C_1 contributed by the sample, and the fringing capacitance, C_e (usually assumed to be a real number), is given by

$$Z_t = \frac{1}{j\omega(C_1 + C_e)} = Z_0 \frac{1 - j\rho_t \tan(\beta x_0)}{\rho_t - j\tan(\beta x_0)} \quad (1)$$

where $j = \sqrt{-1}$

$$C_1 = C_1' - jC_1'' = \varepsilon_r \frac{\pi a^2 \varepsilon_0}{d} = \varepsilon_r C_{0a}$$

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \varepsilon_r'(1 - j\tan\delta)$$

ε_r', and ε_r'' and $\tan\delta$ are the dielectric constant and dielectric loss in the thickness direction of the measured sample, respectively. Equation 1 can also be written as

$$\varepsilon_r' = \frac{\tan(\beta x_0)(\rho_t^2 - 1)}{\omega C_{0a} Z_0[1 + \rho_t^2 \tan^2(\beta x_0)]} - \frac{C_e}{C_{0a}} \quad (2)$$

$$\varepsilon_r'' = \frac{\rho_t[1 + \tan^2(\beta x_0)]}{\omega C_{0a} Z_0[1 + \rho_t^2 \tan^2(\beta x_0)]} \quad (3)$$

where $\omega = 2\pi f$, is the angular frequency, $\beta = \omega/c$ (c is the velocity of light in vacuum, or in air approximately), Z_0 the characteristic impedance of the line, ρ_t the voltage standing wave ratio (VSWR) of termination, and x_0 the distance between the termination and the first standing wave minimum from the termination.

The fringing capacitance, C_e, is generally unknown. When the wavelength $\lambda \gg b - a$ and $d \ll b - a$, there is a good approximate expression for C_e [8]

$$C_e = \frac{4a\varepsilon_0}{\ln[(b - a)/d]} \quad (4)$$

i.e.

$$\frac{C_e}{C_{0a}} = \frac{4d}{\pi a \ln[(b - a)/d]} \ll 1 \quad (5)$$

Obviously, C_e/C_{0a} can be neglected in this case, the measurement, therefore, is usually accomplished under these conditions. However, the condition $d \ll b - a$ can sometimes cause some trouble. For instance, if $2b = 7.00$ mm, $2a = 3.04$ mm, then the dimensions of the sample should be that thickness $d \ll 2.00$ mm and radius $a = 1.52$ mm. For some materials (e.g. ferroelectrics), it is difficult and inconvenient to prepare such thin samples with exactly the same radius as that of the inner conductor. For very low loss materials, it is unlikely that C_e is a pure real number.

The expression for C_1 only holds when the frequency and the dielectric constant of the measured material are low. In the later discussions, we can show that if $f \leq 2$ GHz, $|\varepsilon_r| \leq 100$ and $2a \leq 3.04$ mm, the expression is basically correct. This requirement restricts the usable frequency range and the magnitude of the dielectric constant.

Thus, Equations 2 and 3 are only suitable for the

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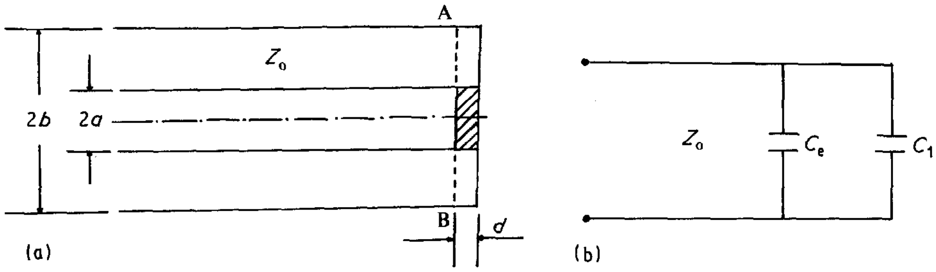

Figure 1 (a) Structural sketch and (b) equivalent circuit of sample holder for Hippel's method.

low microwave frequency measurement of the thinner dielectric samples having small to medium dielectric constants and medium to high losses.

In order to overcome these difficulties, Kolodziej proposed an improved treatment and obtained an expression to solve the complex dielectric constant [1]. However, the expression is too complicated to be used conveniently, and Kolodziej did not discuss the effect of the fringing capacitance, C_e, which cannot be neglected when d is large.

3. The improved lumped capacitance method

Figs 2a and b show a geometrical sketch of the sample holder and its equivalent circuit for the following discussion, respectively. C_2 is contributed by the measured sample, with radius r_1 and thickness d ($r_1 \leqslant a$), and C_3 is contributed by the "fringing" region $r_1 \leqslant r \leqslant b$. $0 \leqslant z \leqslant d$ (including the fringing capacitance). Obviously, if C_1 and C_e in Equation 1 are replaced by C_2 and C_3, respectively, Equation 1 still holds

$$\frac{1}{j\omega C} = \frac{1}{j\omega(C_2 + C_3)} = Z_0 \frac{1 - j\rho_t \tan(\beta x_0)}{\rho_t - j\tan(\beta x_0)} \quad (6)$$

Where $C_2 = C_2' - jC_2''$, $C_3 = C_3' - jC_3''$, the former is dependent on ε_r but the latter is not. $C = C_2 + C_3 = C' - jC''$. C_2 and C_3 will be discussed below.

When d is small (e.g. $d < 2r_1$), it is reasonable to assume that the electric field in the capacitance gap is only in the z direction. Then, it may be shown that $\boldsymbol{E}$ is only dependent on r in the cylindrical coordinate, i.e.

$$\boldsymbol{E}(r, t) = E_z(r) \exp(j\omega t)\boldsymbol{z}_0 \quad (7)$$

where $\boldsymbol{z}_0$ is a unit vector. According to Helmholtz's equation

$$\nabla^2 \boldsymbol{E} + (\beta\varepsilon_r^{1/2})^2 \vec{\boldsymbol{E}} = 0 \quad (8)$$

the solution is

$$E_z = AJ_0(\beta\varepsilon_r^{1/2} r) \quad (0 \leqslant r \leqslant r_1) \quad (9)$$

In terms of Maxwell's equation, the magnetic field is given by

$$\begin{aligned} \vec{\boldsymbol{H}}(r, t) &= \left(-\frac{1}{j\omega\mu_0}\nabla\right)\vec{\boldsymbol{E}}(r, t) \\ &= -A\frac{\beta\varepsilon_r^{1/2}}{j\omega\mu_0} J_1(\beta\varepsilon_r^{1/2} r) \exp(j\omega t)\boldsymbol{\phi}_0 \\ &= H_\phi(r) \exp(j\omega t)\boldsymbol{\phi}_0 \end{aligned} \quad (10)$$

where A is a constant, and J_0 and J_1 are the zeroth and first Bessel functions, and $\boldsymbol{\phi}_0$ is an unit vector in the polar angle direction. The electrical current flowing through the cross-section of the sample is given by

$$I = \oint_{r=r_1} \vec{\boldsymbol{H}} \cdot d\vec{l} = 2\pi r_1 H_\phi(r_1) \exp(j\omega t) \quad (11)$$

So, the impedance at the edge of the sample is

$$\begin{aligned} Z &= \frac{1}{j\omega C_2} = \frac{V}{I} = \frac{E(r_1, t)d}{2\pi r_1 H(r_1, t)} \\ &= \frac{\omega J_0(\beta\varepsilon_r^{1/2} r_1)\mu_0 d}{j2\pi\beta\varepsilon_r^{1/2} r_1 J_1(\beta\varepsilon_r^{1/2} r_1)} \end{aligned} \quad (12)$$

i.e.

$$\begin{aligned} C_2(\varepsilon_r) &= \frac{2\pi\varepsilon_0 r_1 J_1(\beta\varepsilon_r^{1/2} r_1)\varepsilon_r^{1/2}}{d\beta J_0(\beta\varepsilon_r^{1/2} r_1)} \\ &= \frac{\varepsilon_0\varepsilon_r \pi r_1^2}{d}[1 + \tfrac{1}{2}(\tfrac{1}{2}\beta\varepsilon_r^{1/2} r_1)^2 + \tfrac{1}{3}(\tfrac{1}{2}\beta\varepsilon_r^{1/2} r_1)^4 + \ldots] \end{aligned} \quad (13)$$

In principle, C_3 can also be found theoretically, but when the condition $d \ll b - a$ is not satisfied, it is very difficult to find a good approximate expression for C_3, so C_3 had better be found experimentally.

The basic assumption is that when f and d are constant, C_3 is unaffected by the presence of a sample in the sample holder. Let x_0, ρ_t and x_0', ρ_t' represent the measured quantities in the presence and absence of the sample, respectively, noting that when the sample is removed from the sample holder, the "material"

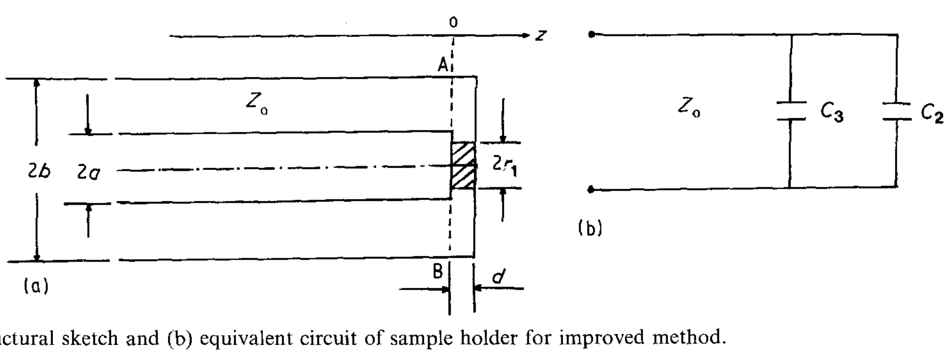

Figure 2 (a) Structural sketch and (b) equivalent circuit of sample holder for improved method.

remaining in the gap is air or vacuum approximately ($\varepsilon_r = 1$), then, using Equations 6 and 13, we obtain

$$C_{02} = C_2(\varepsilon_r = 1) = \frac{2\pi\varepsilon_0 r_1 J_1(\beta r_1)}{d\beta J_0(\beta r_1)}$$

$$= \frac{\varepsilon_0 \pi r_1^2}{d}[1 + \tfrac{1}{2}(\tfrac{1}{2}\beta r_1)^2 + \tfrac{1}{3}(\tfrac{1}{2}\beta r_1)^4 + \ldots] \tag{14}$$

$$C_3' = \frac{\tan(\beta x_0')(\rho_t'^2 - 1)}{\omega Z_0[1 + \rho_t'^2 \tan^2(\beta x_0')]} - C_{02} \tag{15}$$

$$C_3'' = \frac{\rho_t'[1 + \tan^2(\beta x_0')]}{\omega Z_0[1 + \rho_t'^2 \tan^2(\beta x_0')]} \tag{16}$$

$$C_2'(\varepsilon_r', \varepsilon_r'') = \frac{1}{\omega Z_0}\frac{\tan(\beta x_0)(\rho_t^2 - 1)}{[1 + \rho_t^2 \tan^2(\beta x_0)]} - C_3' \tag{17}$$

$$C_2''(\varepsilon_r', \varepsilon_r'') = \frac{1}{\omega Z_0}\frac{\rho_t[1 + \tan^2(\beta x_0)]}{[1 + \rho_t^2 \tan^2(\beta x_0)]} - C_3'' \tag{18}$$

Using Equations 13 to 18 we can obtain ε_r' and ε_r'' by solving the transcendental equations.

Equations 13 to 18 can be used over quite broad frequency and dielectric constant regions. However, when the frequency is higher than 1 GHz, high order mode patterns will probably occur and affect the measurement. Fortunately, Kolodziej has proposed a so-called "calibration" method which is based on the properties of a microwave four-terminal network [1], with which the frequency region to which Equations 13 to 18 can be applied may be between 0.3 and 18 GHz, and the dielectric constant may be up to an order of 10^2 or more. This is especially useful in the measurement of ferroelectrics.

If $r_1 = a$, then

$$\frac{C_2'}{C_1'} = 1 + \tfrac{1}{2}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^2 \frac{\cos(2\delta)}{\cos^2(\delta)} + \tfrac{1}{3}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^4 \frac{\cos(3\delta)}{\cos^3(\delta)}(\delta) + \ldots \tag{19}$$

$$\frac{C_2''}{C_1''} = 1 + 2[\tfrac{1}{2}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^2] + 2\left[\tfrac{1}{3}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^4 \frac{\sin(3\delta)}{\cos(\delta)\sin(2\delta)}\right] + \ldots \tag{20}$$

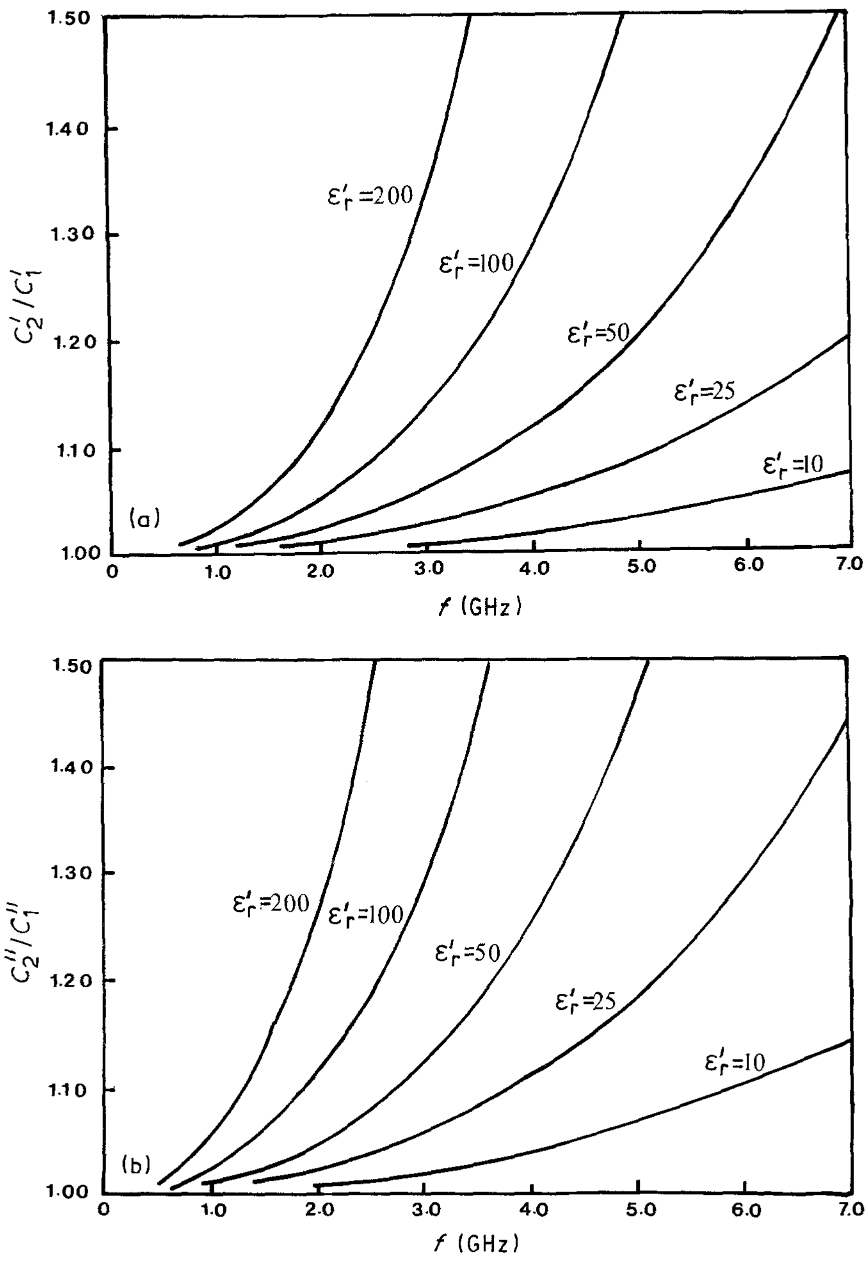

Figure 3 (a) C_2'/C_1' and (b) C_2''/C_1'' curves ($r_1 = a = 1.52$ mm).

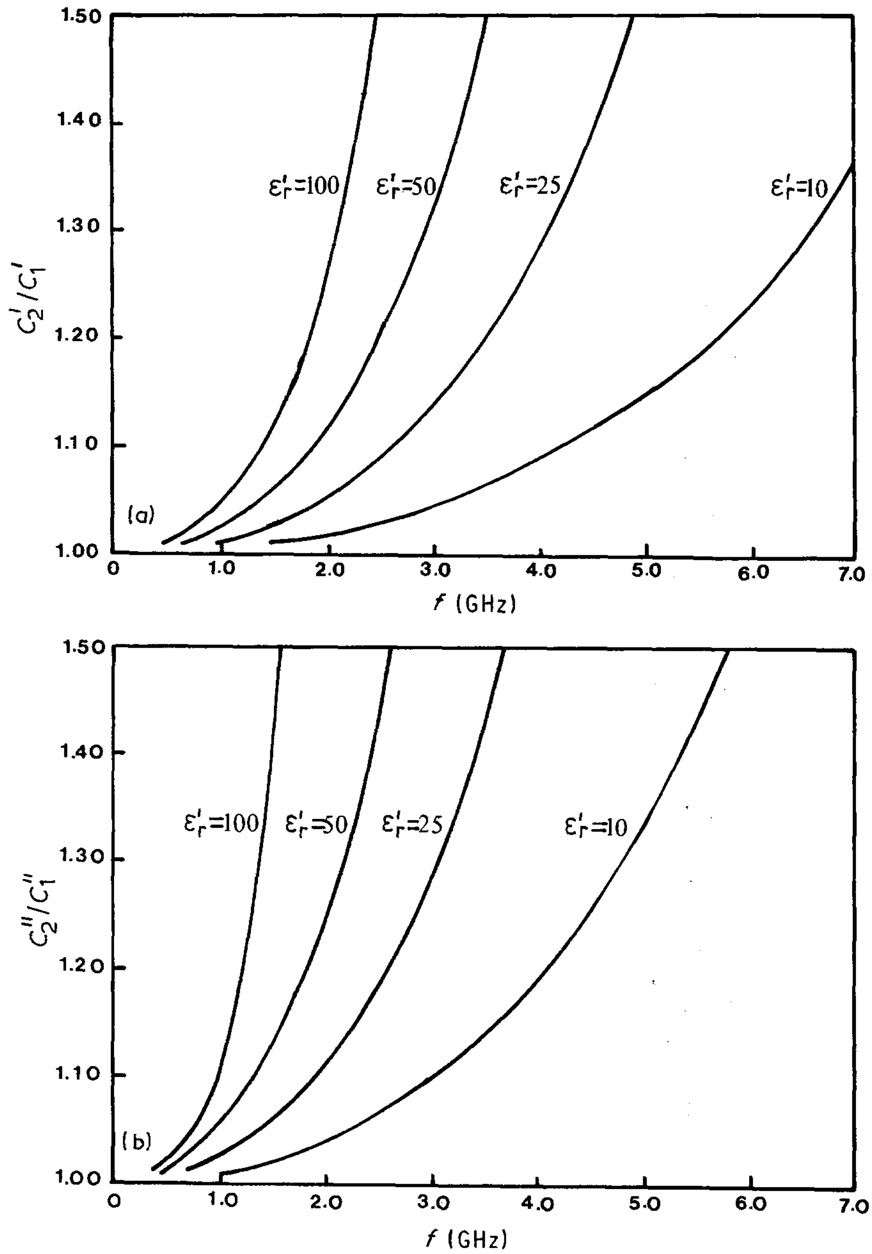

Figure 4 (a) C_2'/C_1' and (b) C_2''/C_1'' curves ($r_1 = a = 3.04\,\text{mm}$).

thus, only when $(\frac{1}{2})\beta\varepsilon_r'^{1/2}a \ll 1$, which means low frequency, small dielectric constant and small radius of sample, $C_2 \approx C_1$. The difference between C_1 and C_2 increases with increase of the quantity $(\frac{1}{2})\beta\varepsilon_r'^{1/2}a$.

When $\tan\delta \ll 1$ and $[(\frac{1}{2})\beta\varepsilon_r'^{1/2}a]^{10} \ll 1$, we may have $\cos(n\delta) \approx 1$, $\sin(n\delta) \approx n\delta$ (for the case when n is not very large), and consequently obtain the following results

$$\frac{C_2'}{C_1'} \approx 1 + \tfrac{1}{2}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^2 + \tfrac{1}{3}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^4 + \tfrac{11}{48}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^6 + \tfrac{19}{120}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^8 \quad (21)$$

$$\frac{C_2''}{C_1''} \approx 1 + (\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^2 + (\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^4 + \tfrac{11}{12}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^6 + \tfrac{19}{24}(\tfrac{1}{2}\beta\varepsilon_r'^{1/2}a)^8 \quad (22)$$

when $\tan\delta = 0$, C_2'/C_1 is the same as Equation 21 and $C_2'' = C_1'' = 0$.

Figs 3 and 4 show the curves of C_2'/C_1' and C_2''/C_1'' for the conditions when the loss is small, $r_1 = a = 1.52\,\text{mm}$, and $r_1 = a = 3.04\,\text{mm}$, from which we can clearly see the changing trends of C_2'/C_1' and C_2''/C_1'' with a, ε_r' and f.

Assume that $(\frac{1}{2})\beta\varepsilon_r'^{1/2} \ll 1$, then, Equations 13 and 14 can be simplified to

$$C_2(\varepsilon_r) = \frac{\varepsilon_0\pi r_1^2}{d}\varepsilon_r = C_{0r}\varepsilon_r \quad (23)$$

$$C_{02} = \frac{\varepsilon_0\pi r_1^2}{d} = C_{0r} \quad (24)$$

Using Equations 15 to 18, 23 and 24 we obtain

$$\varepsilon_r' = \frac{C_2'}{C_{0r}} = \frac{1}{\omega Z_0 C_{0r}}\left[\frac{\tan(\beta x_0)(\rho_t^2 - 1)}{1 + \rho_t^2\tan^2(\beta x_0)} - \frac{\tan(\beta x_0')(\rho_t'^2 - 1)}{1 + \rho_t'^2\tan^2(\beta x_0')}\right] + 1 \quad (25)$$

$$\varepsilon_r'' = \frac{C_2''}{C_{0r}} = \frac{1}{\omega Z_0 C_{0r}}\left\{\frac{\rho_t[1 + \tan^2(\beta x_0)]}{1 + \rho_t^2\tan^2(\beta x_0)} - \frac{\rho_t'[1 + \tan^2(\beta x_0')]}{1 + \rho_t'^2\tan^2(\beta x_0')}\right\} \quad (26)$$

TABLE I Experimental results of KDP (x-direction). Dimensions of sample: $r_1 = 1.385$ mm, $d = 1.940$ mm; vacuum capacitance of sample: $C_{0r} = \varepsilon_0 \pi r_1^2/d = 2.751 \times 10^{-14}$ F

	f (MHz)			
	500	600	800	990
C_2' (10^{12} F)	1.224	1.232	1.234	1.235
C_2'' (10^{15} F)	1.148	1.225	1.231	1.232
C_3' (10^{12} F)	0.158	0.182	0.144	0.127
C_3'' (10^{15} F)	2.251	2.353	3.808	4.763
C' (10^{12} F)	1.382	1.414	1.378	1.362
C'' (10^{15} F)	3.399	3.578	5.039	5.995
ε_{rt}'	44.5	44.8	44.9	44.9
ε_{rf}'	50.2	51.4	50.1	49.5
$\Delta\varepsilon_r'/\varepsilon_r'$	−12.8%	−14.7%	−11.6%	−10.2%
$\tan\delta_t$ (10^4)	9.38	9.95	9.97	9.98
$\tan\delta_f$ (10^4)	24.61	25.28	36.56	44.02
ε_r' [2]	44.3*			
$\tan\delta$ (10^4) [2]	< 5*			
ε_r' [10]				42.5 ± 1.5†

*f = 100 MHz.
†f = 980 MHz.

TABLE II Experimental results of KDP (z-direction). Dimensions of sample: $r_1 = 1.380$ mm, $d = 0.930$ mm; vacuum capacitance of sample: $C_{0r} = \varepsilon_0 \pi r_1^2/d = 5.696 \times 10^{-14}$ F

	f (MHz)			
	500	600	800	990
C_2' (10^{12} F)	1.233	1.240	1.238	1.243
C_2'' (10^{15} F)	0.795	0.510	0.696	0.495
C_3' (10^{12} F)	0.215	0.247	0.211	0.189
C_3'' (10^{15} F)	2.370	1.775	2.605	2.585
C' (10^{12} F)	1.448	1.487	1.449	1.432
C'' (10^{15} F)	3.165	2.285	3.301	3.080
ε_{rt}'	21.6	21.8	21.7	21.8
ε_{rf}'	25.4	26.1	25.4	25.1
$\Delta\varepsilon_r'/\varepsilon_r'$	−17.6%	−19.7%	−17.1%	−15.1%
$\tan\delta_t$ (10^4)	6.46	4.11	5.63	3.99
$\tan\delta_f$ (10^4)	21.88	15.37	22.82	21.54
ε_r' [2]	20.2*			
$\tan\delta$ (10^4) [2]	< 5*			
ε_r' [10]				20.0 ± 0.5†
$\tan\delta$ (10^4) [10]				5†

*f = 100 MHz.
†f = 980 MHz.

which corresponds to the low frequency, low dielectric constant and small sample radius. For instance, if $f \leqslant 2$ GHz, $|\varepsilon_r| \sim \varepsilon_r' < 100$, $r_1 = 1.5$ mm, Equations 23 to 26 hold.

The treatment has eliminated the restriction that $d \ll b - a$ and $r_1 = a$, consequently reducing the difficulty in preparation of samples. However, in order to minimize the errors of the measurement, the ratio d/r_1 should be as small as possible.

When the sample is a high loss material ($\tan\delta > 10^{-1}$), the second term in Equation 26 can be ignored compared with the first term, i.e. C_3 may be assumed to be a real number. But for low or very low loss materials, the second term cannot be neglected.

From the above points it is seen that a more detailed treatment has overcome the disadvantages of the previous method.

4. Results and discussion

Measurement was undertaken only in the low microwave frequency region 0.5 to 0.99 GHz because of the limits of measuring system. A block diagram of the measuring system is given in Fig. 5.

The sample holder was made accurately, with dimensions $2b = 7.00$ mm, $2a = 3.04$ mm. The dielectric material is KDP (KH_2PO_4), and the dimensions of the sample were diameter $2r_1 = 2.70$ to 3.0 mm, thickness $d = 0.90$ to 2.00 mm, subject to experimental restrictions and material availability.

The measurements were made at room temperature. After samples were prepared and surfaces polished, a layer of gold was deposited on each surface of the sample in order to reduce the errors in measurement [9].

Tables I to IV give the experimental results. ε_{rt}', $\tan\delta_t$ and ε'_{rf}, $\tan\delta_f$ represent the results in two cases in which C_3 has been considered and neglected, respectively, and the relative error is defined as $\Delta\varepsilon_r'/\varepsilon_r' = (\varepsilon_{rt}' - \varepsilon_{rf}')/\varepsilon_{rt}'$.

KDP is a dielectric material having a relatively large dielectric constant. The results for ε_{rt}' in the x and z directions are very consistent with those in the literature and the consistency shows the validity of the improved method; however, results for ε_{rf}' are greater than those previously published, which confirms Iskander's predictions [11]. The relative error $\Delta\varepsilon_r'/\varepsilon_r'$ is larger due to greater d.

According to the experimental results, we can show that for a given sample the larger the thickness d, the greater the relative error $\Delta\varepsilon_r'/\varepsilon_r'$, which shows that it becomes more and more important to consider the effect of C_3'. For instance, in Tables I and III, $\Delta\varepsilon_r'/\varepsilon_r'$ is 12% and 7% or so, respectively; in Tables II and IV $\Delta\varepsilon_r'/\varepsilon_r'$ is 17% and 25% or so, respectively.

For samples with nearly the same dimensions, the smaller the dielectric constant, the larger the relative error $\Delta\varepsilon_r'/\varepsilon_r'$. This indicates that when the method is used to measure small dielectric constant, it is very important to consider the influence of C_3' on the experimental data, otherwise the error may be very large.

The order of magnitude of C_2'' is the same as or even smaller than that of C_3'' because KDP is a very low loss material. Therefore, for very low loss material the

$$\boxed{\text{Generator}} \rightarrow \boxed{\text{Isolator}} \rightarrow \boxed{\text{Attenuator}} \rightarrow \boxed{\text{Coaxial Line}} \rightarrow \boxed{\text{Sample Holder}}$$

$$\downarrow$$

$$\boxed{\text{VSWR Indicator}}$$

Figure 5 A block diagram of the measuring system.

TABLE III Experimental results of KDP (x-direction). Dimensions of sample: $r_1 = 1.325$ mm, $d = 0.955$ mm; vacuum capacitance of sample: $C_{0r} = \varepsilon_0 \pi r_1^2/d = 5.114 \times 10^{-14}$ F

	f (MHz)			
	500	600	800	990
C_2' (10^{12} F)	2.253	2.252	2.252	2.251
C_2'' (10^{15} F)	3.971	4.274	3.933	4.070
C_3' (10^{12} F)	0.178	0.189	0.156	0.142
C_3'' (10^{15} F)	3.613	3.027	3.173	3.823
C' (10^{12} F)	2.431	2.441	2.408	2.393
C'' (10^{15} F)	7.584	7.301	7.106	7.893
ε_{rt}'	44.1	44.0	44.0	44.0
ε_{rf}'	47.5	47.7	47.1	46.8
$\Delta\varepsilon_r'/\varepsilon_r'$	−7.7%	−8.4%	−7.0%	−6.4%
$\tan\delta_t$ (10^4)	17.61	18.99	17.48	18.09
$\tan\delta_f$ (10^4)	31.22	29.93	29.50	32.98
ε_r' [2]	44.3*			
$\tan\delta$ (10^4) [2]	< 5*			
ε_r' [10]				42.5 ± 0.5†

*f = 100 MHz.
†f = 980 MHz.

experiment showed that it is no longer correct to neglect C_3''. However, the values of C_2'', C_3'' and C'' are only approximate, due to the large experimental error in determining ρ_t and ρ_t', resulting from their high values (up to 10^2 or 10^3 order of magnitude). The reasons for this are as follows. According to Equations 16, 18, 25 and 26, if ρ_t and ρ_t' are all large, and $\rho_t^2 \tan^2(\beta x_0)$, $\rho_t'^2 \tan^2(\beta x_c') \gg 1$ (corresponding to the measurement for low loss material at low frequency), considering only ρ_t and ρ_t' as variables, we approximately have

$$C_2'' \propto \frac{1}{\rho_t \tan^2(\beta x_0)} - \frac{1}{\rho_t' \tan^2(\beta x_0')} \tag{27}$$

$$C_3'' \propto \frac{1}{\rho_t'} \tag{28}$$

TABLE IV Experimental results of KDP (z-direction). Dimensions of sample: $r_1 = 1.390$ mm, $d = 1.940$ mm; vacuum capacitance of sample: $C_{0r} = \varepsilon_0 \pi r_1^2/d = 2.771 \times 10^{-14}$ F

	f (MHz)			
	500	600	800	990
C_2' (10^{12} F)	0.596	0.604	0.593	0.604
C_2'' (10^{15} F)	0.595	0.602	0.590	0.567
C_3' (10^{12} F)	0.156	0.160	0.146	0.141
C_3'' (10^{15} F)	2.903	2.951	3.728	3.791
C' (10^{12} F)	0.752	0.764	0.739	0.745
C'' (10^{15} F)	3.498	3.553	4.318	4.358
ε_{rt}'	21.5	21.8	21.4	21.8
ε_{rf}'	27.1	27.6	26.7	26.9
$\Delta\varepsilon_r'/\varepsilon_r'$	−26.0%	−26.6%	−24.8%	−23.4%
$\tan\delta_t$ (10^4)	9.99	9.97	9.95	9.39
$\tan\delta_f$ (10^4)	46.58	46.46	58.36	58.47
ε_r' [2]	20.2*			
$\tan\delta$ (10^4) [2]	5*			
ε_r' [10]				20.0 ± 0.5†
$\tan\delta$ (10^4) [10]				5†

*f = 100 MHz.
†f = 980 MHz.

$$\varepsilon_r' \propto \frac{1}{\tan(\beta x_0)} - \frac{1}{\tan(\beta x_0')} \tag{29}$$

$$\varepsilon_r'' \propto C_2'' \tag{30}$$

$$\left|\frac{\Delta C_3''}{C_3''}\right| \propto \left|\frac{\Delta\rho_t'}{\rho_t'}\right| \tag{31}$$

$$\left|\frac{\Delta C_2''}{C_2''}\right| \propto \left|\frac{\Delta\varepsilon_r''}{\varepsilon_r''}\right| \propto \left|\frac{\Delta\tan\delta}{\tan\delta}\right|$$

$$\propto \left|\frac{-\Delta\rho_t/\rho_t^2\tan^2(\beta x_0) + [\Delta\rho_t'/\rho_t'^2\tan^2(\beta x_0')]}{1/\rho_t\tan^2(\beta x_0) - [1/\rho_t'\tan^2(\beta x_0')]}\right|$$

$$\sim \left|\frac{\Delta\rho_t}{\rho_t}\right| \text{ or } \left|\frac{\Delta\rho_t'}{\rho_t'}\right| \tag{32}$$

Clearly, the relative errors of C_3'', C_2'' and $\tan\delta$ or ε_r'' are approximately proportional to those of ρ_t or ρ_t', but ε_r' is independent of ρ_t and ρ_t'. In terms of measurement principle of the VSWR, the higher ρ_t or ρ_t', the larger $|\Delta\rho_t/\rho_t|$ or $|\Delta\rho_t'/\rho_t'|$. In our measurements, $|\Delta\rho_t/\rho_t|$ or $|\Delta\rho_t'/\rho_t'|$ could be up to 50% or more for the ρ_t or ρ_t' having values of 10^2 to 10^3 order of magnitude although the accuracy of the VSWR indicator is high; therefore, the values of C_2'', C_3'' and $\tan\delta$ can only be approximate. Nevertheless, the most of the values for $\tan\delta_t$ in these tables are still qualitatively consistent with those given in the literature, but the results for $\tan\delta_f$ are in poor agreement. This shows that the method is still basically valid for the very low loss materials in spite of the difficulty in accurately determining the VSWR. If we can further increase the accuracy of measurement of the VSWR, then the loss can be expected to be measured with greater precision.

5. Conclusion

An improved lumped capacitance method has been discussed. The improved method can be used theoretically for the dielectric measurement over a wide range of frequency and dielectric constants, avoiding some of the disadvantages of the previous techniques. The method has been proved experimentally to be feasible in the low microwave frequency range 0.5 to 0.99 GHz with very good accuracy in measurement of ε_r', and qualitatively (i.e. in order of magnitude), the measured values of very low loss of ferroelectric KDP are consistent with those given in the literature. We have also shown that the capacitance C_3 contributed by the sample holder itself cannot be considered to be a pure real number in the case of very low loss materials. It can be expected that the accuracy in measurement of the loss of materials with $\tan\delta \geqslant 10^{-3}$ will be much better than at present because the higher $\tan\delta$, the lower the VSWR, and consequently the smaller $|\Delta\rho/\rho|$ or $|\Delta\tan\delta/\tan\delta|$ become, according to the previous analysis. If the accuracy in determining the VSWR is further improved, it will be possible to measure very low loss with a much greater accuracy. Further experiments should be performed in order to confirm the feasibility of the improved method in the higher frequencies and wider complex dielectric constant range.

References

1. H. KOLODZIEJ, "Dielectric Relaxation in Ferroelectrics of the Order-Inorder Type", in "Dielectric and Related Molecular Processes, Vol. 2. A Specialist Periodical Report" (The Chemical Society, Burlington House, London, 1975) pp. 249–87.
2. A. R. VON HIPPEL, "Dielectric Materials and Applications" (M.I.T. Press, Cambridge MA, 1954).
3. S. S. STUCHLY, A. KRASZEWSKI and M. A. STUCHLY, "Uncertainties in Radiofrequency Dielectric Measurement of Biological Substances". *IEEE Trans. Instrum. and Meas.* **IM-36** (1) (1987) 67.
4. T. W. ATHEY, M. A. STUCHLY and S. S. STUCHLY, "Measurement of Radiofrequency Permittivity of Biological Tissues with an Open-End Coaxial Line – part I" *IEEE Trans. Microwave Theory Tech.* **MTT-30** (1982) 82.
5. M. A. RZEPECKA and S. S. STUCHLY, "A Lumped Capacitance Method for the Measurement of the Permittivity and Conductivity in the Frequency and Time Domain – A Further Analysis" *IEEE Trans. Instrum. and Meas.* **IM-24** (1) (1975) 27.
6. S. S. STUCHLY, M. A. RZEPECKA and M. F. ISKANDER, "Permittivity Measurements at Microwave Frequencies Using Lumped Elements" *IEEE Trans. Instrum. and Meas.* **IM-23** (1) (1974) 56.
7. W. L. BELLAMY, S. O. NELSON and R. G. LEFFLER, "Development of a Time-Domain Reflectometry System for Dielectric Properties Measurement" *Trans. of the ASAE,* **28(4)** (1985) 313.
8. H. E. GREEN, "The Numerical solution of Important Transmission Line Problems" *IEEE Trans. on Microwave Theory and Tech.* **MTT-13** (5) (1965) 676.
9. M. A. STUCHLY and S. S. STUCHLY, "Coaxial Line Reflection Methods for Measuring Dielectric Properties of Biological Substances at Radio and Microwave Frequencies – A-Review" *IEEE Trans. on Instrum. and Meas.* **IM-29** (3) (1980) 176.
10. LANDOLT-BORNSTEIN, "Numerical Data and Functional Relationships in Science and Technology, Vol. 2 Band 2", (Springer-Verlag, Berlin, 1969) pp. 68–69.
11. M. F. ISKANDER and S. S. STRUCHLY, "Fringing Field Effect in the Lumped Capacitance Method for Permittivity Measurement" *IEEE Trans. on Instrum. and Meas.* **IM-27** (1) (1978) 107.

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